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Dom- Total Chromatic Number on Various Classes of Graphs

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ABSTRACT: Let G be a graph. For a given χ -total colouring of a graph G, a dominating set S ⊆ V (G) is said to be a dom-total colouring set if it contains atleast one vertex of each colour class of G. The dom-total chromatic number of a graph G is the minimal cardinality taken over all its dom-total colouring sets and is denoted by $\gamma_{d\text{-}tc}$ (G). In this paper, we introduce algorithms to obtain the dom- total colouring and dom-total chromatic number of various classes of graphs.

KEYWORDS: Dom- total chromatic number, central graph of cycle , central graph of jelly fish, tree and complete graph.

I. INTRODUCTION

All graphs considered in this paper are finite, undirected graphs and we follow standard definition of graph theory as found in [1]. Let G = (V, E) be a graph. The open neighborhood N(v) of a vertex v ∈V(G) consists of the set of all vertices adjacent to v. The closed neighborhood of v is N[v] = N (v)∪{v}. An induced subgraph G[S], where S of a graph G is a graph formed from a subset S of the vertices of G and all of the edges connecting pairs of vertices in S. A graph in which every pair of vertices is joined by exactly one edge is called complete graph. A complete bi partite graph is a graph whose vertices can be partitioned into two subsets $V_1$and V_2 such that no edge has both end points in the same subset, and each vertex of V_1 is connected to every vertex of V_2 and vice -verse. A star graph S_n is the complete bipartite graph $K_{1,n-1}$ (A tree with one internal node and n-1leaves).

The path and cycle of order n are denoted by P_n and C_n respectively. For any two graphs G and H, we define the cartesian product, denoted by G×H , to be the graph with vertex set V(G)×V(H) and edges between two vertices (u_1 ,v_1) and (u_2,v_2) iff either $u_1=u_2$ and v_1v_2∈E (H) or u_1u_2∈E(G) and $v_1=v_2$.

A subset S of V is called a dominating set if every vertex in V-S is adjacent to atleast one vertex in S. The dominating set is minimal dominating set if no proper subset of S is a dominating set of G. The domination number γ is the minimum cardinality taken over all minimal dominating set of G. A γ-set is any minimal dominating set with cardinality γ.

A proper coloring of G is an assignment of colors to the vertices of G such that adjacent vertices have different colors. The minimum number of colors for which there exists a proper coloring of G is called chromatic number of G and is denoted by χ (G). A dominator coloring of G is a proper coloring of G in which every vertex of G dominates atleast one color class. The dominator chromatic number is denoted by χ_d (G) and is defined by the minimum number of colors needed in a dominator coloring of G .This concept was introduced by Ralucca Michelle Gera in 2006[2].

In a proper coloring C of G, a color class of C is a set consisting of all those vertices assigned the same color. Let C^1 be a minimal dominator coloring of G. We say that a color class $c_i \in C^1$ is called a non-dominated color class (n-d color class) if it is not dominated by any vertex of G. These color classes are also called repeated color classes. For a given χ -total colouring of a graph G, a dominating set S ⊆ V (G) is said to be a dom- total colouring set if it contains atleast one vertex of each colour class of G. The dom-total chromatic number of a graph G is the minimal cardinality taken over all its dom-total colouring sets and is denoted by $\gamma_{d\text{-}tc}$ (G). Let G be an undirected graph with no loops and parallel edges. The graph formed by subdividing each edge exactly once and joining all the non -adjacent vertices of the graph is called the central graph. It is denoted by the symbol C(G).

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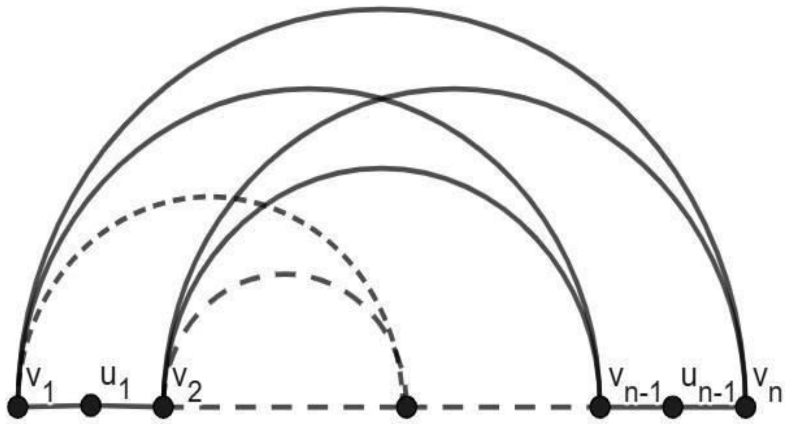
EXAMPLE:



II. MAIN RESULTS

THEOREM 1:
The dom-total chromatic number for central graph of cycle of length n is 6.
$(ie)\ \gamma_{d-tc}(C(C_n)) = 6$

Proof:
Let the central graph is denoted by C_n.
The vertex set $V(C_n) = \{ v1, v2, \ldots, vn\}$ and edge set $E(C_n) = \{e1, e2, \ldots en\}$ where $ei = vi, vi+1$, $1 \le i \le$ n-1 and $en = vn, v1$. The central graph of length n is denoted by $C(C_n)$.
The central graph $C(C_n)$ is formed by subdividing each edge $ei = vi, vi+1, 1 \le i \le$ n-1 of C_n exactly once by adding a new vertex ui and subdividing $en = vn, v1$, by u_n and joining v_i with $vj, 1 \le i, j \le n$, i≠j.
The new vertex set formed is $V(C(C_n)) = \{V1 \cup U1\}$ where $V1 = \{v1, v2, \ldots, vn\}$ and$U1 = u1, u2, \ldots, un$. The new edge set formed is $E(C(C_n)) = \{E_1 \cup E_2\}$ such that $E_1 = \{e1', e2', \ldots, en'\}$ where $ek = vi, vj$, $1 \le i, j \le n$, $k = 1,2,3, \ldots, n$ and $i \ne j$ and $E2 = \{e1'', e''2, \ldots, e''n\}$ where $ei'' = ui, vi+1$.
Define a mapping $\varphi: V(C(Cn)) \to \{1,2,3,4\}$ such that $\varphi(v1) = 1$, $\varphi(v2) = \varphi(v3) = 2$, $\varphi(v4) = \varphi(v5) = 3$, $\varphi(v6) = \varphi(v7) = 4$ and $\varphi(u1) = 4$,$\varphi(u2) = \varphi(u3) = \varphi(u4) = \varphi(u5) = \varphi(u6) = 1$,$\varphi(u7) = 3$.Clearly it is a proper dom-total coloring on vertices and $\gamma_{d-tc}((((n)) \ge 4$. Suppose assume that φ((((n)) = 5 by some optimal dom-total coloring β. Then the dom-total coloring β assigns distinct colors to higher degree non adjacent vertices. Therefore, colors on $v1$, $v2$, $v4$, $v7$ must be distinct.
Now, the fifth color must appear on any of the remaining vertices. If this happens, then there exist any one pair of color with non-adjacent vertices does not have edge between them which is contradiction. So we have $(C(Cn)) \le 4$. Therefore, from the inequalities we arrive the result. Hence the dom-chromatic number for central graph of cycle of length n is 2.
We have $\gamma_{d-t}(G) = \chi(G) + \gamma(G) - 1$. Then $\gamma_{d-}(C(C_n)) = \chi(C(C_n)) + \gamma(C(C_n)) - 1$
Therefore, $\gamma_{d-}(C(C_n)) = 4 + 3 - 1 = 6$.
Illustration:

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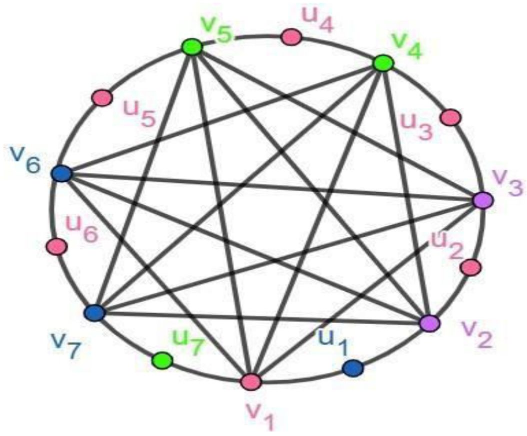
THEOREM 2:



The dom-total chromatic number for central graph of jelly fish graph 4. $(ie)\gamma_{d-tc}(C(J(m, n)))$=4.

Proof:

Let the jelly fish graph is denoted by (m, n). The vertex set be $(J(m, n)) = \{x1, x2, x3, x4 \cup u1, u2, \dots, um \cup v1, v2, \dots, vn\}$ and the edge set be E(J(m, n) = $\{(x1, x2), (x2, x3),(x3, x4),(x4, x1), (x1, x3) \cup (x4, ui) \cup (x2, vj)\}$ where $i = 1,2,3, \dots, n$ and $j = 1,2,3, \dots, m$.

The central graph of jelly fish graph is denotrd by the symbol $(J(m, n))$. The central graph of $(J(m, n))$ is formed by subdividing each edge $(xi, xi+1)$, $1 \le i \le n$ exactly once by adding a new vector ci and joining xi with $xi+2$, $i = 1,2,3,4$ and inclusion of vertex $c5$ between the edge $(x1, x3)$. Also subdividing the pendent vertices connected by $x4$ to pi and $x2$ to qj where= 1,2,3, . . , m and j = 1,2,3, . . , n.

The new vertex set formed is $(C(J(m, n))) = \{x1, x2, x3, x4 \cup c1, c2, c3, c4, c5 \cup pi \cup ui \cup qi \cup vj\}$. The new edge set formed is E (((J(m, n))) ={(x1, c1), (c1, x2), (x2, c2), (x3, c3), (c3, x4), (x4, c4), (c4, x1), $(x1, c5)$, $(c5, x3) \cup (x4, pi) \cup (pi,ui) \cup (x2, qj) \cup (qj, vj)\}$,where $i = 1,2,3, \dots, n$ and $j = 1,2,3, \dots, m$.

Define the mapping φ: $V\ (C(J(m, n))) \rightarrow \{1,2,3\}$ such that $\varphi(x1) = 1$, $\varphi(x2) = 2$, $\varphi(x3) = 3$.

The remaining vertices can be assigned any one of these colors with the proper dom-coloring. Clearly it is a proper dom-coloring on vertices and hence we have $\gamma_{d-tc}(C(J(m, n))) \geq 3$.

Suppose assume that $\varphi\ ((J(m, n))) = 4$ by some optimal coloring μ. Then the coloring μ assigns different colors to higher degree non adjacent vertices. Therefore colors on $x1, x2, x3$ must be distinct.

Now, the fourth color must appear on any of the remaining vertices. If this happens, then there exists any one pair of color which does not have edge between them which is a contradiction to our assumption. So we have $\psi\ (C(J(m, n))) \leq 3$.

Hence from the inequalities we arrive the required result.

Hence the chromatic number for central graph of jelly fish graph is 3.$(ie)\gamma_{d-ct}\ (C(J(m, n)))$ =3.

We have $\gamma_{d-tc}\ ((J(m, n))) = \chi\ (C(J(m, n))) + \gamma\ (C(J(m, n))) - 1$

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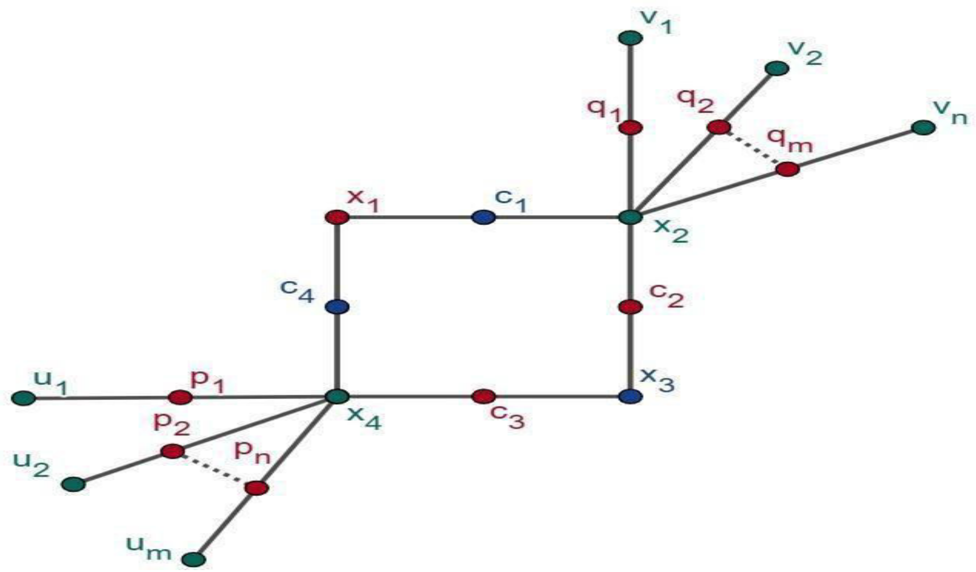
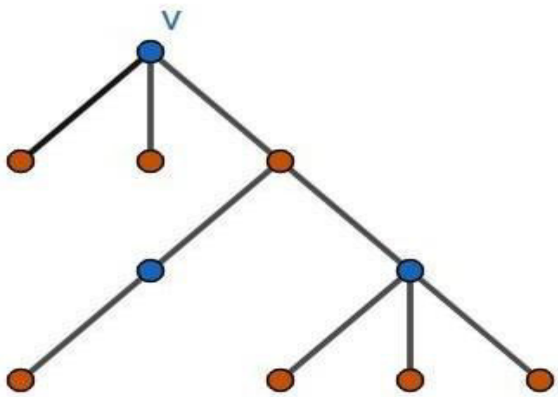
Therefore $_{\{d\text{–tc}\}}\,(C(J(m,n))) = 3 + 2 - 1 = 4$.



THEOREM 3:
Every tree with atleast 2 vertices has dom-chromatic number 2.
ie) $\gamma_{\text{d-tc}}(T) = 2$.
PROOF:
Let T be a tree with $(T) \geq 2$. Let $v \in (T)$.
Consider T be rooted at v. Let v be colored with color 1. Let the neighbours of v be colored with color 2. Let the neighbour of those vertices be colored with color 1. Continue the process of coloring.
In the tree there is a unique path between any 2 vertices along any path in T. The vertices alternate colors so no pair of adjacent vertices receive the same color. This is 2-colouring.
Therefore, $(T) = 2$.

Therefore, Every tree with atleast 2 vertices is 2-chromatic graph.
We have $\gamma_{d\text{–t}}(T) = \chi(T) + \gamma(T) - 1$.
Therefore, $\gamma_{d\text{–t}}(G) = 2 + 1 - 1 = 2$



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THEOREM 2.4:
$\gamma_{d\text{-tc}}(G) = 1\ iff\ G$ is just a single vertex(K_1).
PROOF:
Assume $\gamma_{d\text{-tc}}(G) = 1$.
To show that G is just a single vertex.
There is one vertex v that dominates the whole graph and v is adjacent to every other vertex, hits every color class in some proper coloring. Then there can be only one color class .
This implies that $(G) = 1$ means no edges,but v is adjacent to every other vertex the only possibility is there must be no other vertices.
Hence G must be exactly K_1 Conversely,
Assume that G is K_1.
To prove that $\gamma_{d\text{-ct}}(G) = 1$. Therefore, $(G) = 2$.
It is obvious that one vertex dominates itself and hits its only color class.
Therefore, $_{d\text{-tc}}(G) = 1$

THEOREM 5:
For the complete bipartite graph $K_{m,}$ with $2 \leq m \leq n$, $\gamma_{d\text{-tc}}(K_{m,n})$ =2
PROOF:
The dominating set number is $(K_{m,n}) = 2$, one vertex from each part dominates the whole graph.
The total chromatic number is $(K_{m,n}) = 2$, the two parts are the two color classes.
Let S be the set of size of 2 consisting of one vertex from each parts such as S is a dominating set and hits both colors. Thus S is a dominating 2 −coloring set, so $\gamma_{d\text{-tc}}(K_{m,n}) \leq 2$. No set of size 1 can be a dom-total chromatic set, so $\gamma_{d\text{-tc}}(K_{m,n}) = 2$.

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